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Stabilization of Networked Control Systems with Access Constraints and Delays

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***Abstract*— This paper discusses the stabilization of linear networked control systems (NCSs) in which the medium connecting plant and controller imposes access constraints and known delays. We apply a communication protocol which "decouples" the stabilization problem, in the sense that the medium access policy can be selected independently of the controller while jointly stabilizing the NCS. Our approach is based on reducing the infinity of possible stabilizing communication policies down to a set of sequences which preserve the NCS's controllability and observability. We extend previously established results by treating delays and medium access constraints simultaneously and by demonstrating how a controller designed for a delay-free NCS can be re-used when delays are present. We include a numerical experiment that illustrates our approach.**

I. INTRODUCTION

This work is a continuation of recent efforts [21], [22] to address the problem of stabilizing Networked Control Systems (NCSs). Such systems, which – as their name suggests – close their feedback loops via networks or other shared communication media, have enjoyed steadily-rising interest among researchers during the last decade. In part because of their potentially flexible architecture, NCSs can now be found in domains ranging from academic laboratories to commercial vehicle and aerospace applications (see, for example, [5] for a broad range of related articles).

From a wide perspective, NCSs research has thus far focused on systems which are subject to one of three main types of communication constraints: limitations on the bit rate at which controller and plant may communicate [19], [17], [13], medium access constraints (i.e., limitations on the number of sensors and actuators that can communicate with the controller simultaneously) [3], [6], [22], [20], and transmission delays [14], [10], [7].

In this paper we are specifically interested in the stabilization of linear NCSs in which delays and medium access constraints are present simultaneously. In that setting, one must specify not only a controller but also a communication policy which will govern data exchanges between controller and plant. Previous works on NCS with transmission delays have often assumed that the controller was designed a-priori, and investigated the effect of delays on the stability of the resulting closed loop (e.g., [18], [1]; see also [23] for an informative review). Medium access constraints were typically not considered. Other approaches have explored the effect of multiple delays in NCSs [8] and necessary and sufficient conditions for NCS stability in the presence of delays [12].

This paper presents a method for designing a controller and communication policy (in the sense of [6]) that jointly stabilize a linear NCS at a desired decay rate. Our approach builds on [22] by modeling NCSs with medium access constraints and delays as equivalent linear time-varying (LTV) systems whose dynamics combine those of the underlying plant together with the choice of communication policy.

The difficulty of arriving at a solution to the NCS stabilization problem, turns out to depend both on the communication protocol that governs plant-controller interactions as well as on the NCS model adopted at the outset. The importance of the latter choice is highlighted by the fact that, depending on the approach taken, the question of whether a stabilizing feedback controller exists could be NP-hard — even if the communication sequence is fixed in advance (see [6] and references therein) — or much simpler [22]. The solution described here will have the effect of "decoupling" the choice of controller from that of the communication policy. This is rather fortunate and is usually not the case in problems involving the control of NCSs.

In the next Section we show how a linear plant subject to medium access constraints and transmission delays can be modeled as a linear time-varying (LTV) system of the same dimension. We will use periodic communication sequences [3], [6] to schedule access for the various sensors and actuators in a way that preserves the plant's reachability and observability; this scheduling will take place off-line. Section III, discusses the construction of an observer-based controller with delay compensation using results from linear systems theory. An example is given in Section IV.

II. NCS WITH MEDIUM ACCESS CONSTRAINTS AND TRANSMISSION DELAYS

Consider an NCS in which a remote controller interacts with a linear time-invariant (LTI) plant via a shared medium (see Fig. 1) which limits communication in two ways: i) it imposes communication delays, and ii) it cannot support

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simultaneous communication with all of the plant's sensors and actuators. We will assume that the underlying plant

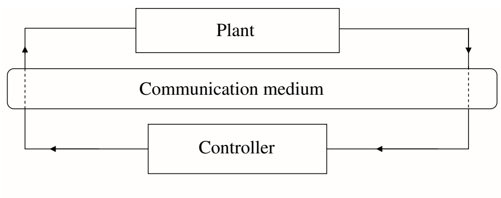

Fig. 1. A Networked Control System

evolves in continuous time:

$$\dot{x} = A_c x + B_c u; \quad y = C_c x, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^p$ are the plant's state, input, and output, respectively. The output is sampled periodically at $t = kT,\ k = 1, 2, ...,$ and is sent to the controller, subject to the following constraints.

- The communication medium has w_σ available *output channels*, where $1 \le w_\sigma < p$. Therefore, only w_σ of the p sensors can transmit their output to the controller at any one time, while others must wait. Furthermore, data transmitted by the plant at time t arrives at the controller at $t + \tau_{pc}$, for some $\tau_{pc} > 0$.
- Similar constraints apply at the plant's input stage, where m actuators share w_ρ *input channels*, ($1 \le w_\rho < m$) to receive control signals from the controller. At most w_ρ of the m actuators can access the input channels simultaneously, while controller-to-plant communication is subject to a delay of $\tau_{cp} > 0$. The delays τ_{cp} and τ_{pc} will be taken to be known and fixed.

Based on these assumptions, the NCS takes on the configuration shown in Fig. 2, where the "open" or "closed" status of the switches indicates the medium access status of the corresponding sensors or actuators. The vectors $\bar{y} \in \mathbb{R}^p$

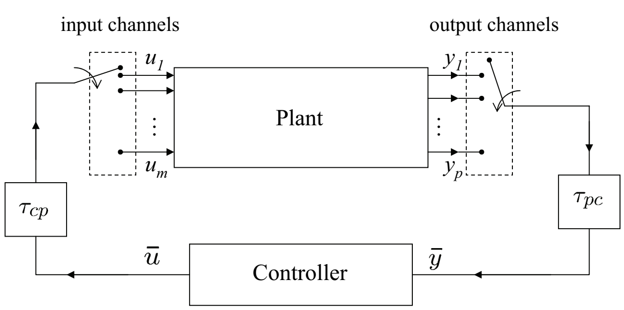

Fig. 2. Modeling medium access constraints and transmission delays

and $\bar{u} \in \mathbb{R}^m$ are the input and output of the controller, respectively, and will generally differ from the plant output y and input u (in a manner which will be made precise shortly) because of the communication constraints outlined above. The remainder of this paper is concerned with the following problem.

Problem 1: Find a medium access policy for the plant's sensors and actuators, and a feedback controller that exponentially stabilize the NCS, given the access constraints (w_ρ, w_σ) and delays (τ_{pc}, τ_{cp}) it must operate under.

A. Modeling the Communication Constraints

Upon receipt of a set of output data $\bar{y}$, the controller computes and transmits a control $\bar{u}$, which will reach the plant τ_{cp} time units later. Unless $\tau_{cp} + \tau_{pc}$ is a multiple of the sampling period T, an input will arrive at the plant "between" samplings. It will be convenient to arrange matters so that input data arriving during $(kT, kT + T]$ will be "buffered" at the plant's input stage and will be applied to the actuators at the next sampling instant, $kT + T$. A timing diagram is shown in Fig. 3. This assumption can be easily lifted but we will

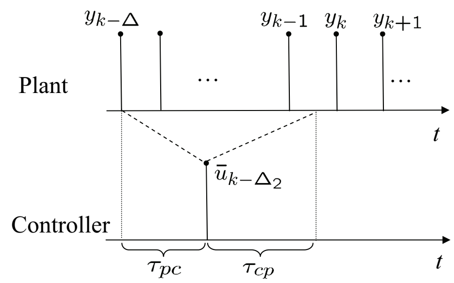

Fig. 3. Timing diagram. Plant outputs arrive at the controller τ_{cp} time units after transmission. The controller computes and transmits $\bar{u}(k)$, which arrives at the plant with a delay of τ_{cp} and is applied in the Δ_2-th sampling period following transmission. Here, $\Delta_2 = \lceil \tau_{pc}/T \rceil$, $\Delta = \lceil (\tau_{cp} + \tau_{pc})/T \rceil$.

make use of it here because it simplifies the notation. We also assume that the controller computes inputs instantaneously; if that is not the case, τ_{cp} can be increased to include the time needed for computation.

We will let $u(k)$ denote the input that arrived the plant some time in $(kT, kT + T]$, while $x(k)$ and $y(k)$ will be the samples of the state and output, respectively, at $t = kT$. Likewise, $\bar{y}(k)$ will be the output data that arrived at the controller during $(kT, kT + T]$. Inputs produced by the controller will be indexed by the time interval in which they were computed, i.e., $\bar{u}(k)$ will be the input produced by the controller during $(kT, kT + T]$. Because plant inputs are buffered and outputs are sampled periodically, the controller will generate precisely one such input per sampling interval.

Under the protocol described above, the plant becomes an LTI sampled-data system:

$$x(k+1) = Ax(k) + Bu(k); \quad y(k) = Cx(k), \tag{2}$$

where (A, B, C) is the discretized version of the plant (A_c, B_c, C_c) with sampling period T, and by slight abuse of notation we have written $x(k), y(k)$ and $u(k)$, in place of $x(kT), y(kT)$ and $u(kT)$, respectively.

We now focus on the communications "bottleneck" which occurs due to the lack of simultaneous access by the plant's sensor's and actuators. For all $i = 1, \cdots, p$, let $\sigma_i(k) : \mathbb{Z} \mapsto \{0, 1\}$ denote the medium access status of sensor i

during $(kT, kT+T]$, where 1 means "accessing" and 0 means "not accessing". The access status of all p sensors will be represented by the p-to-w_σ *"communication sequence"* [3], [6], $\sigma(k) = [\sigma_1(k), \cdots, \sigma_p(k)]^T$.

Definition 1: An M-to-N communication sequence is a map, $\sigma(k) : \mathbb{Z} \mapsto \{0,1\}^M$, satisfying $\|\sigma(k)\|^2 = N$, $\forall k$.

B. NCS Dynamics

In any NCS communication protocol, there is a choice to be made on what a receiver should do when an output (resp. input) is unavailable because the corresponding sensor (actuator) cannot obtain medium access. A common practice [3], [6] is zero-order holding (ZOH), which means that each time an actuator looses its connection to the controller, it retains its input level until communication is re-established. This choice carries intuitive appeal but may introduce time-varying delays in the closed-loop dynamics, thereby increasing the complexity of Problem 1 [6]. Instead, some NCS stabilization problems [20], [22] (including Problem 1) become much simpler if the missing data is simply ignored by the controller and plant. Forgoing the ZOH will lead us to a solution which is complete and which decouples the choice of controller from that of the communication sequences.

Let $\rho(k)$ be the $w_\rho - to - m$ communication sequence that governs medium access for the plant's actuators. An input $u_i(k)$, will be available for transmission to the plant only when actuator i is accessing the communication medium, i.e., $\rho_i(k) = 1$. At other times ($\rho_i(k) = 0$) it is assumed that a *zero* value will be used by the actuator, while the corresponding input $\bar{u}_i(k)$ produced by the controller will be ignored due to it being unavailable. Define the quantities $\Delta_1 \triangleq \lceil \tau_{pc}/T \rceil$, $\Delta_2 \triangleq \lceil \tau_{cp}/T \rceil$, $\Delta \triangleq \Delta_1 + \Delta_2$. Based on the previous discussion,

$$u_i(k) = \rho_i(k - \Delta_2)\bar{u}_i(k - \Delta_2), \tag{3}$$

for all $i = 1, 2 \cdots m$, because an input arriving at the plant during $[kT, kT+T)$ was produced Δ_2 sampling intervals ago, and the choice of which actuators to address was made by the controller at the time of transmission. It will be convenient to define the *matrix form*[1] of a communication sequence $\eta(k)$, by $M_\eta(k) \triangleq \text{diag}(\eta(\text{k}))$, so that

$$u(k) = M_\rho(k - \Delta_2)\bar{u}(k - \Delta_2). \tag{4}$$

Similarly, whenever sensor j loses its access to the communication medium, the controller sets $\bar{y}_j = 0$ until sensor j regains medium access; then, the output $\bar{y}$ used by the controller is related to the "true" plant output by:

$$\bar{y}(k) = M_\sigma(k - \Delta_1)y(k - \Delta_1). \tag{5}$$

From (2), (4) and (5) we see that from the controller's point of view, the plant acts as a time-varying system with input $\bar{u}$ and output $\bar{y}$ (Fig. 4):

$$\begin{aligned} x(k+1) &= Ax(k) + BM_\rho(k - \Delta_2)\bar{u}(k - \Delta_2), \\ \bar{y}(k) &= M_\sigma(k - \Delta_1)Cx(k - \Delta_1), \end{aligned} \tag{6}$$

[1]One could eliminate the zero rows of $M_\rho(k)$ (columns of $M_\sigma(k)$) in order to arrive at a controller of lower dimensions, $w_\rho \times w_\sigma$, as in [20].

and that the effects of medium access constraints and delays have been captured by the terms $M_\sigma(k)$ and $M_\rho(k)$. We will refer to (6) as the "extended plant"; its state coincides with that of the original plant, hence the NCS can be stabilized by designing a feedback controller that stabilizes (6). We can

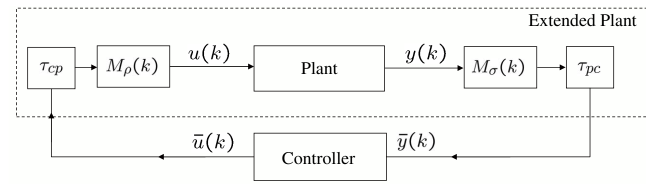

Fig. 4. The extended plant

now re-cast Problem 1 in a more precise form.

Problem 2: Find a pair of communication sequences $\sigma(k), \rho(k)$ and a feedback controller that exponentially stabilize the NCS (6), given the medium access constraints (w_ρ, w_σ) and delays (τ_{pc}, τ_{cp}).

Remark: Our protocol requires that actuators, in a sense, "turn off" when they are not communicating, and may result in inputs with large step changes. If that is not desirable, one could amend the extended plant model to include ZOH for inputs, by augmenting the state to include those inputs which are *not* updated, i.e.,

$$\begin{aligned} x(k+1) &= Ax(k) + BM_\rho(k - \Delta_2)u_{ZOH}(k), \\ u_{ZOH}(k+1) &= u_{ZOH}(k)(I - M_\rho(k - \Delta_2)) + \\ &\quad + M_\rho(k - \Delta_2)\bar{u}(k - \Delta_2). \end{aligned} \tag{7}$$

The discussion that follows can be modified accordingly if one wishes to adopt (7). The details are omitted due to space constraints.

III. Stabilization with Medium Access Constraints and Delays

Our choice of communication protocol, and the LTV dynamics that arise as a result, make it possible to solve Problem 2 by selecting the communication sequences independently of the controller. We first "narrow down" the choice of communication to those sequences which preserve the plant's reachability and observability. Then, we can go on to design a delay compensator and observer-based controller for the NCS dynamics that ensue.

A. Selection of the Communication Sequences

The task of designing effective communication sequences is quite challenging, despite their intuitive appeal. Previously proposed methods [4], [2], [15] usually rely on strong assumptions about the plant or involve significant complexity (see [15], [9] for examples of sequence optimization in optimal control problems). Because our objective is stability, we are faced with a simpler situation; instead of optimizing the communication in some way, we will seek to characterize the set of sequences $\rho(\cdot), \sigma(\cdot)$ that guarantee the existence of a stabilizing controller.

If the original plant (A_c, B_c, C_c) is controllable and observable, then it is always possible to select a pair of communication sequences that preserve those properties in the extended plant [21]. Briefly, suppose that $x(0) = 0$, that there are no transmission delays ($\tau_{cp} = \tau_{pc} = 0$), and let the extended plant (6) evolve from $k = 0$ to some $k = k_f$. Then,

$$x(k_f) = R \cdot \left[\begin{array}{cccc} \bar{u}^T(0) & \bar{u}^T(1) & \cdots & \bar{u}^T(k_f - 1) \end{array} \right]^T,$$

where

$$R = [A^{k_f-1}BM_\rho(0), A^{k_f-2}BM_\rho(1), \ldots, BM_\rho(k_f - 1)]. \tag{8}$$

The extended plant (6) will be reachable in $[0, k_f]$ iff $\mathrm{rank}(R) = n$. Recall that for each k, $\rho(k)$ is an m-dimensional vector of w_ρ ones and $m - w_\rho$ zeros. Hence, at each step k, $M_\rho(k)$ has the effect of "selecting" w_ρ columns from the m columns of the term $A^{k_f-k-1}B$ on the RHS of (8). Therefore, finding a sequence that preserves reachability is equivalent to choosing w_ρ columns from each term B, AB, A^2B, etc., so that the resulting R has n independent columns. It is known [21] that this is always possible if A and B meet certain conditions:

Definition 2 ([16]): A discrete-time linear system is called l-step reachable (observable) if $l > 0$ is an integer such that the system is reachable (observable) on $[i, i + l]$ for any i.

Theorem 1 ([21]): Let A be invertible and the pair (A, B) be reachable. For any integer $1 \le w_\rho < m$, there exist integers $l, N > 0$ and an N-periodic m-to-w_ρ communication sequence $\rho(\cdot)$ such that the extended plant (6) is l-step reachable.

A similar result holds for observability, and there is an upper bound $k_f \le \left\lceil \frac{n}{w_\rho} \right\rceil$ (resp. $k_f \le \left\lceil \frac{n}{w_\sigma} \right\rceil$) on the period of $\rho(\cdot)$ and $\sigma(\cdot)$. It is easy to show that the above results hold without changes when transmission delays *are* present. A column selection algorithm for constructing reachability and observability-preserving sequences is given in [21], and repeated here for convenience:

1) Let $\Gamma_i = [A^{ni+n-1}B, A^{ni+n-2}B, \cdots, A^{ni}B]$, where $i = 1, 2, \ldots$.
2) Let $L_i = \{\gamma_i^0, \cdots, \gamma_i^{n-1}\}$ be any n linearly independent columns from Γ_i.
3) Let $L = L_0$. Replace γ_0^1 in L with a column from L_1 while maintaining $\mathrm{rank}(L) = n$.
4) For $i = 2, \cdots, n - 1$, replace γ_0^i in L with a column from L_i while keeping the rank of L fixed.

The resulting L has one column from each Γ_i ($i = 0, \cdots, n - 1$) and has rank n.

B. Observer-based Feedback with Delay Compensation

Rewrite the extended plant as

$$\begin{aligned} x(k+1) &= Ax(k) + \bar{B}(k - \Delta_2)\bar{u}(k - \Delta_2), \\ y(k) &= \bar{C}(k)x(k), \end{aligned} \tag{9}$$

where we have defined $\bar{B}(k) \triangleq BM_\rho(k)$ and $\bar{C}(k) = M_\sigma(k)C$. For each time k, we will construct a family of predictors (inspired by the delay compensator in [11]) whose states, $\hat{x}(k, i)$, are estimates of the plant's "true" state $x(k - \Delta_1 + i)$. These predictors are to be updated by the controller each time it receives new output data.

- We begin by observing the plant's state, using data which was transmitted to the controller Δ_1 sampling periods in the past,

$$\begin{aligned} \hat{x}(k, 1) = A\hat{x}(k-1, 1) + \bar{B}(k - \Delta)\bar{u}(k - \Delta) + \\ H(k)\left(\bar{y}(k) - \bar{C}(k - \Delta_1)\hat{x}(k-1, 1)\right). \end{aligned} \tag{10}$$

- The state $\hat{x}(k, 1)$ is propagated forward in order to estimate the plant's state at the time the controller's currently-generated output $\bar{u}(k)$ is to reach the plant:

$$\hat{x}(k, j+1) = A\hat{x}(k, j) + \bar{B}(k-\Delta+j)\bar{u}(k-\Delta+j), \tag{11}$$

for $j = 1, ..., \Delta - 1$.

- The estimate $\hat{x}(k, \Delta)$ of the future state $x(k + \Delta_2)$ is used in the feedback control law:

$$\bar{u}(k) = G(k)\hat{x}(k, \Delta). \tag{12}$$

Theorem 2: For the NCS (6) and predictor (11), define the estimation error to be

$$e(k) \triangleq x(k - \Delta_1) - \hat{x}(k - 1, 1). \tag{13}$$

Under the feedback controller (12), the closed loop dynamics of (6) are

$$\left[\begin{array}{c} x(k+1) \\ e(k - \Delta_2 + 2) \end{array} \right] = \mathcal{A}(k) \left[\begin{array}{c} x(k) \\ e(k - \Delta_2 + 1) \end{array} \right], \tag{14}$$

where

$$\mathcal{A}(k) \triangleq \left[\begin{array}{c|c} \begin{array}{l} A+ \\ \bar{B}(k-\Delta_2)G(k-\Delta_2) \end{array} & V(k) \\ \hline 0 & \begin{array}{l} A- \\ H(k-\Delta_2+1)\bar{C}(k-\Delta+1) \end{array} \end{array} \right], \tag{15}$$

and $V(k)$ is a matrix that depends on the (finite) time histories of $\bar{B}, \bar{C}, H$ and G.

To prove Th. 2, we will need the following result:

Lemma 1: For all $k = 1, 2, ...,$ and $r = 1, 2...,$

$$\hat{x}(k, r+1) = \hat{x}(k+r, 1) - \sum_{i=0}^{r-1} A^i H(k+r-i)\bar{C}(k+r-i-\Delta_1)e(k+r-i). \tag{16}$$

Proof: The statement is proved by induction. First, we verify that the statement holds for $r = 1$. From (10) we have

$$\begin{aligned} \hat{x}(k+1, 1) = A\hat{x}(k, 1) + \bar{B}(k+1-\Delta)\bar{u}(k+1-\Delta) + \\ +H(k+1)\bar{C}(k+1-\Delta_1)\underbrace{(x(k+1-\Delta_1) - \hat{x}(k, 1))}_{e(k+1)}, \end{aligned}$$

while (11) implies that

$$\hat{x}(k, 2) = A\hat{x}(k+1, 1) + \bar{B}(k-\Delta+1)\bar{u}(k-\Delta+1).$$

Combining the last two equations, we obtain

$$\hat{x}(k, 2) = \hat{x}(k, 1) - H(k+1)C(k+1-\Delta_1)e(k+1).$$

One can also verify (again using (10) and (11)) that the statement holds for $r = 2$. Assuming now that (16) holds for an index of $r+1$, we show that it also holds for an index of $r + 2$. From (10),

$$\begin{aligned}\hat{x}(k+r+1,1) &= A\hat{x}(k+r,1)\\ &+ \bar{B}(k+r+1-\Delta)\bar{u}(k+r+1-\Delta)\\ &+ H(k+r+1)\bar{C}(k+r+1)e(k+r+1),\end{aligned} \tag{17}$$

while (11) implies that

$$\hat{x}(k,r+2) = A\hat{x}(k,r+1)+\bar{B}(k-\Delta+r+1)\bar{u}(k-\Delta+r+1).$$

Using our hypothesis that (16) holds, the last equation yields

$$\begin{aligned}\hat{x}(k,r+2) &= A\hat{x}(k+r,1)\\ &- A\sum_{i=0}^{r-1} A^i H(k+r-i)\bar{C}(k+r-i-\Delta_1)e(k+r-i)\\ &+ \bar{B}(k-\Delta+r+1)\bar{u}(k-\Delta+r+1).\end{aligned} \tag{18}$$

By comparing (17) and (18), we obtain the desired result:

$$\begin{aligned}\hat{x}(k,r+2) &= \hat{x}(k+r+1,1)\\ &- \sum_{i=0}^{r}\Big(A^i H(k+r+1-i)\bar{C}(k+r+1-i-\Delta_1)\cdot\\ &\qquad e(k+r+1-i)\Big).\end{aligned} \tag{19}$$

■

Proof of Th. 2: From the definition of the estimation error (13) and the observer dynamics (10), it is easy to verify that

$$e(k+1) = (A - H(k)\bar{C}(k-\Delta_1))e(k), \tag{20}$$

which establishes the lower-right block of $\mathcal{A}(k)$ in (15).

Under feedback (12), the NCS evolves according to:

$$\begin{aligned}x(k+1) &= Ax(k) + \bar{B}(k-\Delta_2)\bar{u}(k-\Delta_2)\\ &= Ax(k) + \bar{B}(k-\Delta_2)G(k-\Delta_2)\hat{x}(k-\Delta_2,\Delta).\end{aligned}$$

Using Lemma 1, the last equation yields

$$\begin{aligned}x(k+1) = Ax(k) &+ \bar{B}(k-\Delta_2)G(k-\Delta_2)\cdot\\ &\Big(\hat{x}(k+\Delta_1-1,1)\\ &- \sum_{i=0}^{\Delta-2} A^i\bar{C}(k-i-1)e(k+\Delta_1-1-i)\Big).\end{aligned}$$

By adding and subtracting the term $\bar{B}(k-\Delta_2)G(k-\Delta_2)x(k)$ from the right-hand side of the last equation, and noticing that $e(k+\Delta_1-i-1)$ can be expressed in terms of $e(k+1-\Delta_2)$ by means of (20), we obtain

$$\begin{aligned}x(k+1) &= (A+\bar{B}(k-\Delta_2)G(k-\Delta_2))x(k)\\ &- \bar{B}(k-\Delta_2)G(k-\Delta_2)e(k+\Delta_1)\\ &- \sum_{i=0}^{\Delta-2}\Big(A^i H(k+\Delta_1-1-i)\bar{C}(k-i-1)\cdot\\ &\qquad \prod_{j=k-\Delta_2+2}^{k+\Delta_1} W(j)e(k-\Delta_2+1)\Big),\end{aligned} \tag{21}$$

where $W(j) \triangleq A-H(j)\bar{C}(j-\Delta_1)$. To obtain the top row of (14), (15) we again express $e(k+\Delta_1)$ in terms of $e(k-\Delta_2+1)$ in (21), and define $V(k)$ to be the sum of the factors that multiply $e(k-\Delta_2+1)$ in the resulting equation for $x(k+1)$.

■

C. Gain selection

If the communication sequences $\rho(\cdot)$, $\sigma(\cdot)$ are chosen to be N-periodic, then the closed-loop dynamics (13)-(15) become periodic as well. The block upper-triangular structure of $\mathcal{A}(k)$ suggests that it is sufficient to design the controller and observer gains $G(\cdot)$ and $H(\cdot)$ so that: i) G and H are N-periodic, and ii) $A+\bar{B}(k-\Delta_2)G(k-\Delta_2)$ and $A-H(k-\Delta_2+1)\bar{C}(k-\Delta+1)$ are stable. This is always possible if $\rho(\cdot)$ and $\sigma(\cdot)$ are chosen to preserve the reachability and observability of (A,B,C), as per Theorem 1. The required gain sequences can be computed as follows. For (6), define

$$\begin{aligned}&\mathcal{W}_{\eta\alpha}(k_0,k_f) \triangleq\\ &\quad\sum_{j=k_0}^{k_f-1}(\eta\alpha)^{4(k_0-j)}A^{k_0-j-1}\bar{B}(j)\bar{B}^T(j)(A^{k_0-j-1})^T,\\ &\mathcal{M}_{\eta\alpha}(k_0,k_f) \triangleq\\ &\quad\sum_{j=k_0}^{k_f-1}(\eta\alpha)^{4(j-k_f+1)}(A^{j-k_0})^T\bar{C}^T(j)\bar{C}(j)A^{j-k_0}.\end{aligned}$$

The following is a rephrasing of Theorem 29.5 in [16] for the extended plant (6).

Theorem 3: Let the extended plant (6) be l-step reachable, l-step observable, and A be invertible. Fix $\alpha > 1$ and $\eta > 1$. Then, the feedback and observer gains

$$\begin{aligned}G(k) &= -\bar{B}^T(k)(A^{-1})^T\mathcal{W}_{\eta\alpha}^{-1}(k,k+l),\\ H(k) &= [(A^{-l})^T\mathcal{M}_{\eta\alpha}(k-l+1,k+1)A^{-l}]^{-1}\cdot\\ &\qquad\qquad\cdot(A^{-1})^T\bar{C}^T(k),\end{aligned}$$

are such that the extended plant (6) is uniformly exponentially stable with rate α under the feedback law $\bar{u}(k) = G(k)\hat{x}(k,\Delta)$, where $\hat{x}(k,\Delta)$ is the estimate produced by the observer and delay compensator (10)-(11).

IV. An Example

We simulated an NCS whose plant was an unstable 4-th order continuous-time system with parameters (1):

$$A_c = \begin{bmatrix} -3 & 0.1 & 0 & 0\\ 0.1 & 15/8 & 0 & 0\\ 1 & -0.1 & -5/3 & 1/2\\ 0 & 0 & 0 & 2\end{bmatrix},$$

$$B_c = \begin{bmatrix} 0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix},\ C_c = \begin{bmatrix} 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\end{bmatrix}.$$

The plant's initial condition was set to $x(0) = [0.3, 0.5, 0.7, 0.6]^T$. The communication medium could carry only one of the inputs (resp. outputs) at any time (i.e., $w_\rho = w_\sigma = 1$). Plant outputs were sampled and transmitted with a period of $T = 0.1$s; the transmission delays were $\tau_{pc} = 0.13$s and $\tau_{cp} = 0.28$s. The input and output communication sequences, $\sigma = \{[0,1]^T, [1,0]^T, \cdots\}$ and $\rho = \{[1,0]^T, [0,1]^T, \cdots\}$, were constructed using the column selection algorithm in [21]. It is easy to verify that under this pair the extended plant is 6-step reachable and 6-step observable, by checking the rank of (8).

The resulting NCS was stabilized by the controller described in Sec. III-B. The observer and feedback gains, $H(k)$, $G(k)$, were calculated using the formulas in Th. 3, with $\alpha = 2$ and $\eta = 1.2$:

$$G(2k+1) = \begin{bmatrix} -252.18 & -12.41 & 0.054 & 0.0054 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$G(2k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -57.48 & 0.066 & -62.12 & -12.73 \end{bmatrix},$$

$$H(2k+1) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.0032 & 0.0087 & -0.5 & 2.07 \end{bmatrix}^T,$$

$$H(2k) = \begin{bmatrix} -0.42 & 1.9 & -0.34 & 0.52 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

for $k = 0, 1, 2, ...$ The observer's initial condition was $\hat{x}(0) = [0,0,0,0]^T$. The NCS's state evolution (sampled at $t = T, 2T, ...$) is shown in Fig. 5. The prediction error, $x(k) - \hat{x}(k+\Delta_2)$, showed a similar response, being identically zero for $k > 9$.

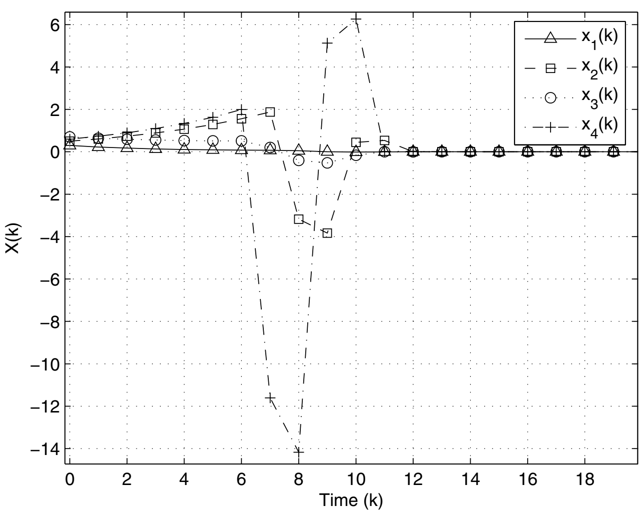

Fig. 5. Closed-loop NCS: state evolution under 2-periodic communication.

V. Conclusions and Future Work

We explored the problem of stabilizing linear NCSs with medium access constraints and transmission delays by applying a delay-compensated feedback controller and an accompanying medium access policy. We proposed an NCS architecture in which: i) plant outputs are sampled periodically and sent to a remotely-located controller, ii) inputs received by the plant are stored and applied at the next sampling time, and iii) the controller and plant use information only from those sensors and actuators which are granted medium access. The assumptions of input buffering and the "removal" of zero-order holding of plant inputs are not essential and can be lifted. Our model makes it possible to design the control and communication policies separately, leading to a straightforward solution of the stabilization problem. Using the notion of a communication sequence as a basic modeling tool, one can consider extensions of this work to NCSs which are subject to "dropped" data packets and random delays, in addition to the constraints discussed here. Problems of that kind, as well as the LQG control of NCS, are the subject of ongoing work.

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