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A Markov-Based Decision Model of Tax Evasion for Risk-Averse Firms in Greece

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Abstract We develop a Markov-based optimization model that captures the process via which a risk-averse firm in Greece decides whether to engage in tax evasion. The firm seeks to maximize the expected utility of its wealth, the latter viewed as a function of the portion of profits which the firm attempts to conceal from the government. Our model takes into account the basic features of the Greek tax system, including random audits and tax penalties applied when the audit reveals any wrongdoing. The proposed model is used to (1) show that the parameters currently in place are conducive to tax evasion and (2) "chart" the problem's parameter space in order to identify "virtuous" combinations (from the point of view of the government), and obtain a relationship between audit probability, tax penalty and likelihood of the firm engaging in tax evasion.

Key words Markov Chains • Optimization • Taxation • Greece

1 Introduction

Greece is currently under severe economic stress, facing perhaps its most serious crisis in its modern history. The government's plan for coping with the country's high debt and budget deficits has included a "rescue package" backed by the ECB and the IMF, combined with a series of austerity measures. One of the most talked-about and widely agreed-upon measures—for which, however, there has been little in the way of implementation—has to do with combating tax evasion, which is openly acknowledged as a sizeable drain on the country's finances, as well as one of its most persistent problems. The purpose of this paper is to explore a Markov-based

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optimization model which may be used to (1) investigate the expected behavior of Greek firms[1] with respect to tax evasion and (2) test candidate tax policies before they are implemented.

In this work we are interested in obtaining a simple and adaptable optimization model that captures the salient features of the Greek tax system and can be used as a decision support tool by the government, as the latter seeks a framework that neither rewards "cheating" nor penalizes firms any more than is necessary to curb tax evasion. We adopt the perspective of a "representative" firm, which is assumed to act selfishly in order to optimize its wealth via tax evasion, by leaving some of its profits unreported. The firm can be viewed as choosing to allocate its profits between two assets (as in [1]). One is risk-free and involves the firm declaring its profit and paying the proper tax, meaning that it ends up with a somewhat lower wealth as a result. The other asset is risky: profit is concealed (zero tax payed); however, the firm's wealth may be reduced in the future (more than if it had behaved honestly) if the firm is audited. In these circumstances, a risk-averse self-interested firm is expected to maximize the expected utility of its wealth by choosing to conceal some fraction of its profits. We wish to find the optimal allocation which the firm will adopt, as a function of its risk aversion level and the parameters of the tax system (audit probability and tax penalty). Furthermore, we would like to identify alternative tax parameters which could lead to higher government revenues by eliminating or reducing the incentive for tax evasion. Towards that end, we "chart" the parameter space and compute a surface which quantifies the firm's incentive to cheat (i.e., its optimal percentage of profits to conceal). The proposed model allows us to estimate the relative efficiency of the tax penalty and audit probability as tax evasion deterrents under different levels of risk aversion.

The remainder of this paper is structured as follows. After a brief literature review, we give a brief description of the Greek tax system in Sect. 2 and formulate a corresponding Markov-based model from the point of view of a risk-averse firm that would like to increase its net wealth by evading taxes but is worried about tax penalties in the event of an audit. We describe the objective function the firm seeks to optimize and discuss our choice of model parameters. Section 3 presents the firm's optimal strategy together with numerical results on its expected behavior and offers a brief discussion of the relevant policy implications.

1.1 Related Work

The approach adopted here is to examine the problem of optimal tax evasion (and by extension, optimal taxation) from the perspective of a self-interested firm. To the best of our knowledge, most of the optimization models aimed at taxation have adopted a macroeconomic viewpoint. Early work on begins with [1] who proposed

[1]The word "firms" refers to incorporated entities in Greece, operating according to the general accounting principles commonly known in Europe as S.A.—Société Anonyme).

the portfolio allocation idea used in this work, but optimized a macroeconomic equilibrium-based model. Subsequent work [2], concentrated on the effects of increased probability of detection, or of tax rates [4]. For an optimal control viewpoint on taxation, see [13]. The criteria based on which tax evasion decisions are made are discussed in [5]. Some authors, e.g., [7, 8] addressed the morality of taxpayers and auditors as variables, or investigated the idea of bonuses to auditors that reveal tax evasion [11].

The model in [15] captured the trade-off between fines and audit probabilities while proposing different treatment for risk-neutral versus risk-averse firms. Other work relevant to our setting includes [14] and [3] who applied Bedford's law to tax evasion and other types of financial fraud. With respect to Greece in particular, the tax evasion literature (e.g., [12] and [16]) provides some good theoretical and empirical discussion but no rigorous analysis with respect to how tax policy should be shaped. A recent exception is [9].

2 Model Description

We proceed to give a brief description of the Greek tax system, leading to the formulation of an optimization problem which the firm is faced with each year.

2.1 The Greek Tax System for Firms

In Greece, firms report their profits at the end of each fiscal year, and pay a tax rate of 24%. Typically, the government does not have adequate information on the firm's true profits, which may be manipulated through a variety of methods. Two of the most often used include (1) manipulation of financial statements to under-report income and (2) invoices (often issued by another, usually short-lived firm) that document supposed expenses and are used to offset profits. To discourage "cheating," firms are subject to random audits, and there are monetary penalties which apply for unreported profits.

A tax audit can reveal a firm's true profit but is costly and resource-intensive. When an audit occurs, it can cover up to a 5-year period in the past, meaning that the government retains the right to audit a tax statement for up to 5 years from its submission. After that 5-year window, any unreported profit, unpaid tax, etc., is essentially capitalized by the firm. Although there are no official data published on the number of audits performed each year, information obtained from the press and tax professionals suggests that the probability of a firm being audited is approximately 5%, with the distibution being skewed towards firms which are approaching the 5-year mark (and therefore a past tax statement which is about to go beyond the reach of the audit process).

When an audit does reveal tax evasion, the penalties imposed depend on the amount of unreported income, and the time elapsed since the offense occurred. The total cost to the firm is the tax originally due on the unreported income, plus a 2% monthly penalty on that tax. Thus, "older" tax evasion decisions are potentially more costly than recent ones. The total penalty amount is subject to a 2/3 discount if the firm agrees to pay promptly once the evasion is discovered. Finally, the penalty amount cannot exceed twice the original tax owed.

The above is, of course, not an exhaustive description of the Greek tax code, but does include the features which are most germane. Some aspects, such as VAT payments (collected via an independent mechanism) are not considered here, but could be incorporated into the model at later stages.

2.2 The Model

We will describe the possible tax status of the firm in any given year via a Markov chain which evolves on a set $\mathscr{S}$ with $\mathscr{S} = \{A, N_1, \ldots, N_4\}$. States in $\mathscr{S}$ are labeled as follows:

- A: the firm is being audited,
- N_j: the firm's last audit was $j = 1, \ldots, 4$ years ago.

These labels will be used mainly to facilitate the discussion below. Otherwise, for notational convenience, we will refer to states in $\mathscr{S}$ by integer, in order of appearance (i.e., $A \to 1, \ldots, N_4 \to 5$).

Let F be the firm's annual profit. Each year, k, the firm decides the fraction $u_k \in [0,1]$ of its profit to conceal (thereby declaring to the government only $(1-u_k)F$), and then transitions to a new state in $\mathscr{S}$, with probabilities $a_{ij} = P(s_k = i | s_{k-1} = j)$, where the indices i and j indicate the ith and jth states in $\mathscr{S}$. Here, we have assumed that the transition probabilities are independent of the firm's actions and that the annual profit is constant. These assumptions can be removed, but we will not take up the issue here, mainly because of space considerations.

Based on the discussion of Sect. 2.1, we can express the Markov chain as $x_{k+1} = Ax_k$, where x_k is the state's probability distribution at time k. The stochastic matrix $A = [a_{ij}]$ is not written down here, but can be easily read off from the transition diagram shown in Fig. 1, where p denotes the overall audit probability (i.e., the fraction of tax returns that the government is able to audit each year). There is little-to-none official data on p; we estimate its value at $p = 0.05$ (and its distribution among states as per Fig. 1), based on reports in the Greek financial press and discussions with individuals familiar with the inner workings of the tax authority and audit mechanism in Greece. As the transition diagram indicates, the probability of an audit is heavily skewed towards firms in the last (N_4) state; there, the firm has been unaudited for 4 years and is about to file its fifth consecutive tax statement. Therefore, if it is not audited in the next time period, the oldest of these five statements will go beyond the reach of any future audits.

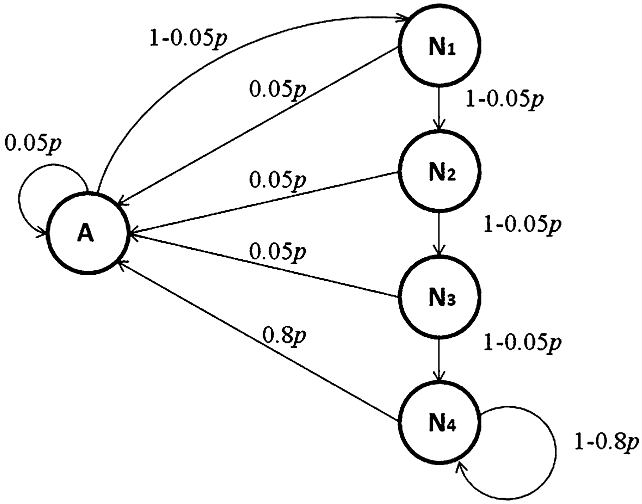

Fig. 1 Markov transition diagram for our simplified model of the Greek tax system as it pertains to firms. The overall probability of a tax audit each year is p, distributed so that 80% of audits involve firms with at least one tax statement whose 5-year statute of limitations is about to elapse

The firm allocates its yearly profits between two "assets," as discussed in Sect. 1. One is a risky asset, $R(j)$, which corresponds to concealing profit when being at the jth state. The payoff in that case is that the firm gets to keep more of its profit (since it pays no tax on the amount concealed), at a risk of being audited sometime in the next five time periods, in which case it will have to pay the tax back, plus a penalty. The alternative to R is the risk-free asset, S whose payoff is the firm's after-tax profit, i.e., $S = F(1-r)$, where $r = 0.24$ is the tax rate. The firm then has a portfolio whose worth at time k depends on the allocation of profits between the two assets:

$$W(u_k, j) = (1-u_k)S + u_k R(j) \quad (1)$$

We note that R (and thus W) depends on the state $j = 1, \ldots, 5$ in which the firm is at when making its decision, because (the probability distribution of) the number of years that will pass until the firm is audited depends on its initial state j.

Based on our description of the Greek tax system, we can specify the risky asset's return rate as follows:

$$R(j) = \begin{cases} F(1-\gamma^n r(1+3/5n\beta)) & \text{the firm is audited in year } k+n,\ n \leq 5 \\ F & \text{no audit before year } k+6. \end{cases} \quad (2)$$

Notice that $R(j)$ (and thus $W(u_k, j)$) is a random variable whose value depends on when the next audit will occur. In (2), $\gamma \in [0,1]$ is a discount coefficient, used to capture the present value of any tax and penalties the firm might pay in the future. The term $-F\gamma^n r$ corresponds to the tax the firm will pay for every percentage of its profit it is found to have concealed, while $-F\gamma^n\beta 3/5$ is the penalty rate.

3 Optimal Firm Behavior

We can now proceed with computing the behavior expected of the firm. We will assume that the latter acts according to a constant relative risk aversion utility function:

$$U(x) = \frac{x^{1-\lambda}}{1-\lambda}, \tag{3}$$

where λ is the "average" firm's risk aversion coefficient. The firm's objective is then to select the fraction of profit, u_k, that it will attempt to hide from the government at time k, in order to maximize the expected utility of its portfolio:

$$\max_u \{\mathbb{E}(U(W(u,j)))\}, \quad j = 1,2,\ldots,5. \tag{4}$$

where for simplicity we have dropped the subscript k in u, and the expectation in (4) is taken with respect to the probability distribution on the number of years $\{1,2,\ldots,5,\infty\}$ from the time the firm makes a decision until the next audit occurs (either within 5 years, or never). This is essentially the distribution of first passage probabilities from each state j of our Markov chain, to the first (audit) state in precisely n steps.

$$f_{1j}^{(n)} = p(s_{k+n} = 1 | s_k = j, s_{k+i} \neq 1 \text{ for } i = 1,\ldots,n-1).$$

It is well known that for finite n, the $f_{ij}^{(n)}$ can be computed by solving the following upper-triangular system of algebraic equations:

$$a_{ij}^{(n)} = \sum_{r=1}^{n} f_{ij}^{(r)} a_{ii}^{(n-r)} \tag{5}$$

where $a_{ij}^{(k)}$ denotes the (i,j)th element the Markov transition matrix after it is raised to the kth power, A^k. Consequently (4) leads to the firm's optimal policy:

$$u^*(j) = \arg\max_u \Bigg\{ \sum_{n=1}^{5} f_{1j}^{(n)} U\left((1-u)(1-r)F + u(1-\gamma^n r(1+3/5n\beta))F\right) + \left(1 - \sum_{n=1}^{5} f_{1j}^{(n)}\right) U\left((1-u)(1-r)F + uF\right) \Bigg\}. \tag{6}$$

It must be noted that in the last equation it is possible for the argument of $U(\cdot)$ to be negative when, for example, u and the penalty coefficient β are sufficiently high (e.g., the penalty is so high that it exceeds the firm's annual profit). This can be dealt with in various ways, but perhaps the simplest one is to notice that there is always a choice of u that results in positive wealth (namely $u = 0$). Thus, since U is continuous and the firm presumably prefers positive wealth to negative wealth we can simply restrict the maximization in (6) to the range of values for u that result in $W(u,j) > 0$.

3.1 Charting the (p,β) Parameter Space

We obtained first-order optimality conditions from (6), and solved them for a range of tax penalty and audit probability values to obtain the firm's optimal tax evasion level in each case. Before examining the firm's behavior, we explain some of our assumptions and justify the parameter ranges that we consider meaningful to explore.

3.2 Parameter Selection

The discount coefficient γ in (6) was based on an assumed interest rate of 3%, i.e., $\gamma = 1/(1+0.03) = 0.9709$. In order to isolate the effect of tax-penalties and audit probabilities on firm behavior, the tax rate r was be kept fixed at its current levels. However, our model can easily be used to examine the effects of changing the tax rate as well.

The firm's profit, F, was estimated based on published data from the Greek Secretariat of Information Systems [10] which indicates that in 2009, the average firm declared approximately €75,700. The true F is, of course, known only to the firm itself. Studies on tax evasion estimate Greece's "hidden economy" at 25–40%, of what is documented, depending on the assumptions used (see, for example [6, 12]), implying a true average profit in the range of $95{,}000$–$106{,}000$.

Regarding the range of values for the fraction of profit, $u \in [0,1]$, which may be hidden, it may be practically impossible for a firm to claim zero profits by overstating expenses and/or hiding income. At least some income will be documented, and the firm may be under pressure to show profits for shareholders or capital markets. To account for this, u could be interpreted in a "marginal" sense, i.e., viewed as the fraction of profits that *can* be concealed; alternatively, one could apply (6) by replacing F with the portion of profits that are concealable and thus at stake in the portfolio. Assuming that $F = 100{,}000$ and that the firm has the option of hiding 30–40% of that amount, the risk aversion coefficient λ that results in the level of tax evasion estimated in the literature must be in the range of 0–12.

The figures estimated above are clearly approximations. Nevertheless, what is presented here could serve as the basis for a decision support model at the hands of official entities which are in a position to have more precise knowledge of the required parameters and can thus "tune" the model appropriately. In the following, we chart the firm's tax evasion behavior for $\lambda = 0$ and $\lambda = 6$.

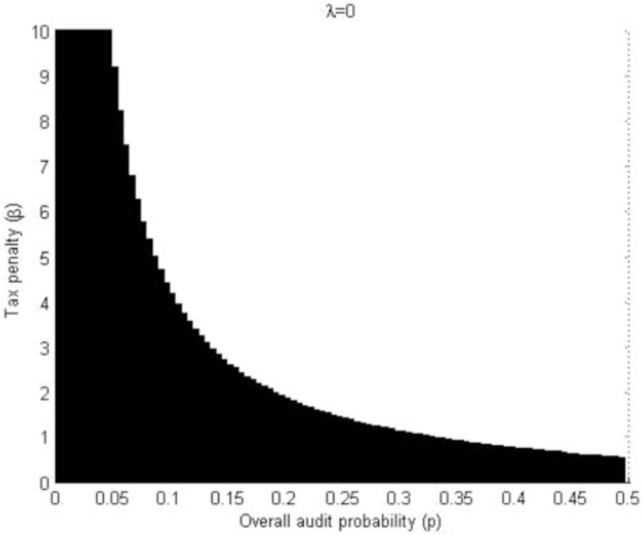

Fig. 2 Tax evasion for a risk-neutral firm. *Dark area*: the firm conceals as much profit as possible ($u^* = 1$). *White area*: it is optimal for the firm to disclose all profit ($u^* = 0$). The non-smoothness of the boundary is due to discretization

3.3 Risk-Neutral Firm ($\lambda = 0$)

We computed the firm's optimal decision u^* in the (β, p) space when $\lambda = 0$, i.e., U is linear and the firm is risk-neutral. Linearity makes the firm's behavior simple to describe, because the solution of (6) is either $u = 0$ or $u = 0$. Figure 2 illustrates the resulting mapping assuming the firm is in the first (audit) state (i.e., $u^*(1)$). The situation is qualitatively similar when the firm is in the other four states.

We observe is a kind of boundary in the p-vs-β space, below which the firm always attempts to hide as much profit as possible (($u^* = 1$, black region). On the other hand, it is best for the firm to disclose all profit ($u^* = 0$) at points above the boundary. Notice the very high tax penalty coefficients required to "induce" honest behavior. At the current $p = 0.05$ audit probability, the tax penalty needs to be approximately 10, which, after the 2/5 discount is applied corresponds to a net tax penalty rate of 600% per annum on unpaid taxes, a rate significantly higher than the baseline 24%. This suggests that tax penalties may be ineffective without the backup of an effective audit mechanism. Finally, a tax penalty rate lower than $\beta = 0.6$ is ineffective in eliminating tax evasion for a risk neutral firm, even for unrealistically high audit probabilities (up to almost 50%).

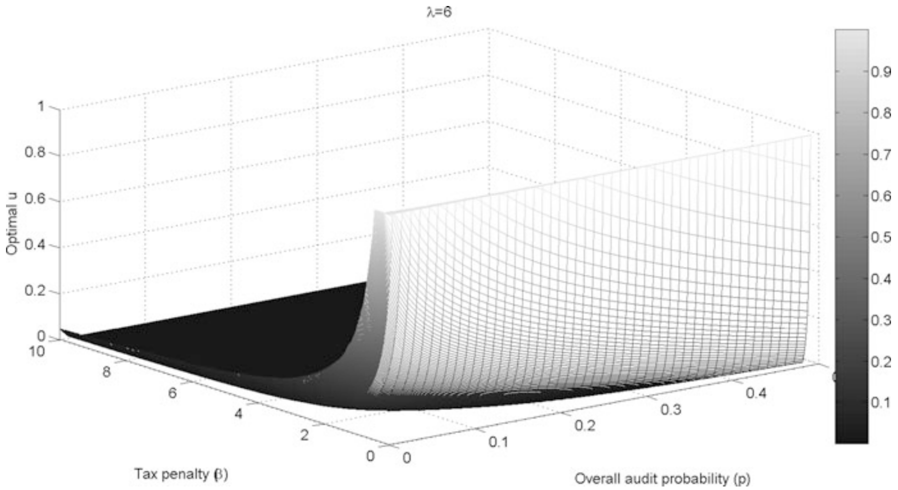

Fig. 3 Tax evasion mapping for a risk-averse firm ($\lambda = 6$) assuming the firm is deciding immediately after an audit. *Gray* levels represent tax evasion between 0% (*dark*) and 100% (*bright*)

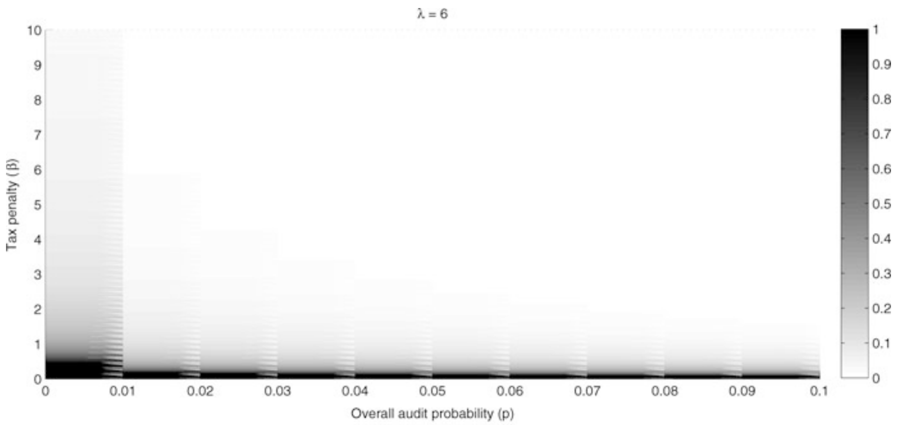

Fig. 4 Top view of figure on the *left*, focusing on audit probabilities between 0.005 and 0.1. *Gray* levels represent tax evasion between 0% (*white*) and 100% (*black*)

3.4 Risk-Averse Firm ($\lambda = 6$)

When the firm is risk averse, its decisions regarding tax evasion are no longer "all-or-nothing." Figures 3 and 4 illustrate the $u^*(1)$ vs. p vs. b surface in the region $\beta \in [0,5]$ and $p \in [0.005, 0.5]$. Other values of λ generate surfaces of similar geometry.

In Fig. 4 the surface is viewed along the u^* axis, with values of u^* encoded in the gray levels, and we have zoomed into the region $\beta \in [0,5]$ and $p \in [0.005, 0.1]$.

The range for p is of practical interest because in the case of audits, for example, a high p is not easy to implement (mode audits require hiring of new personnel, training, etc.). Inspection of the surface for the $\lambda = 6$ case at higher magnification reveals that the current set of parameters ($p = 0.05, \beta = 0.24$) is rather ineffective, and that in order to significantly curb tax evasion (say at under 10%) the tax penalty coefficient needs to be raised, from the current $\beta = 0.24$ to approximately $\beta = 1.5$ when $p = 0.01$, and to $\beta = 0.9$ when $p = 0.05$.

4 Conclusions

We described a Markov-based optimization model for capturing the process by which a risk-averse firm in Greece decides to what degree it will engage in tax evasion. Each year, the firm acts so as to maximize the expected utility of its wealth, based on the first passage probabilities of transitioning to an "audit" state before the statute of limitations on its decision expires. Our model captures the basic features of the Greek tax system, in which a firm is either audited or accumulates up to five unaudited tax statements, and audits can cover up to 5 years in the past. Some of the model's parameters (tax penalties, tax rates, average firm profit) were set based on government reports, while others (audit probabilities, firm's risk aversion) were estimated implicitly from publicly available data and estimates on Greece's hidden economy. The proposed model was used to "chart" the audit probability vs. tax penalty space in order to compute the expected level of tax evasion in which the firm engages. Such charts were produced here for an "average" firm, but could easily be tailored to specific sectors or even individual firms.

The work presented here can be viewed as a basic tool for informing tax policy by elucidating the firm's expected behavior under different scenarios. Opportunities for future work include revisiting the problem where the "average" firm considered in our analysis is replaced by a population of firms with a given distribution for their risk aversion, and augmenting the model to include additional aspects of the tax system, such as VAT payments and closure.

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