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TESTING FOR GRANGER CAUSALITY IN THE PRESENCE OF CHAOTIC DYNAMICS[⊥]

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Abstract:

An increasing number of recent articles applying powerful tests for non-linear causality have fuelled interest in discovering complex dynamics in macroeconomic and financial data. This paper is an attempt to add to the set of available tools by proposing a test for determining the source of causal relationships when a certain class of complex dynamics, that includes chaotic non-linearities, is present.

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KEYWORDS: Complex Dynamics, Granger Causality, Non-linear Causality, Bivariate Noisy Mackey-Glass Model.

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INTRODUCTION

As emphasized in recent work by Kyrtsou and Labys (2006, 2007), and Diebolt and Kyrtsou (2005), the identification of non-linear causal relationships between economic variables is of paramount importance for the quantification and qualification of the true impact of shocks. Non-proportionality, feedback behaviour, and short-term forecastability are some of the properties of non-linear systems that make the use of non-linear analysis so attractive for the interpretation of inherent economic dynamics. When non-linear causality is identified, there is a strong possibility that a small variation in one variable can have multiplicative and non-proportional effects on the others. If, additionally, this non-linear causal dependence is driven by a positive feedback mechanism, the impact of the shock may be reinforced.

An increasing number of recent articles applying powerful tests for non-linear causality (including the Baek and Brock test (1992), and its modifications by Hiemstra and Jones (1994), and Diks and Panchenko (2005)) reveal a high level of interest in discovering related structure in macroeconomic and financial data. Nevertheless, existing methods for detecting non-linear causality do not allow for a concurrent characterization of the underlying dynamics. It would be useful to have available non-linear causality test(s) that are specific to those dynamic structures which are deemed relevant for real economic data. In this paper, we attempt to move in that direction by proposing a test for determining the source of causal relationships when complex dynamics, such as chaotic non-linearities, are present. The paper is organized as follows. In section 1, we describe our method for identifying causality relationships generated by a specific bivariate noisy chaotic model which has special relevance for applied economic analysis. Section 2 includes a pair of numerical experiments and discusses the empirical results.

1. TEST FOR NON-LINEAR CAUSALITY

The causality test proposed in this paper assumes an underlying process with a special type of non-linear structure, known as the *bivariate noisy Mackey-Glass model* (Kyrtsou and Labys, 2006, 2007). The model is as follows:

$$X_t = \alpha_{11}\frac{X_{t-\tau_1}}{1+X^{c_1}_{t-\tau_1}} - \delta_{11}X_{t-1} + \alpha_{12}\frac{Y_{t-\tau_2}}{1+Y^{c_2}_{t-\tau_2}} - \delta_{12}Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N(0,1) \tag{1}$$

$$Y_t = \alpha_{21}\frac{X_{t-\tau_1}}{1+X^{c_1}_{t-\tau_1}} - \delta_{21}X_{t-1} + \alpha_{22}\frac{Y_{t-\tau_2}}{1+Y^{c_2}_{t-\tau_2}} - \delta_{22}Y_{t-1} + u_t \quad u_t \sim N(0,1),$$

where the α_{ij}, and δ_{ij} are parameters to be estimated, τ_i are delays, and c_i are constants. The model (1) can produce various types of dependencies by adjusting the parameters c_i and τ_i. The principle advantage of (1) over simple VAR alternatives is that the non-linear Mackey-Glass (hereafter M-G) terms are able to capture more complex dependent dynamics in a time series. As discussed in Kyrtsou and Labys (2006, 2007) the identification of significant M-G terms in a pair of series suggests a non-linear feedback law as the generating mechanism of the interdependences between *X* and *Y*.

The proposed causality test attempts to detect whether past samples of a variable *Y* have a significant non-linear effect (of the type $\frac{Y_{t-\tau_2}}{1+Y_{t-\tau_2}^{c_2}}$) on the current value of another variable, *X*. Algorithmically, the test is similar to the linear Granger causality test, except that the two models fitted to the *X* and *Y* series are M-G processes. We begin by estimating the parameters of a M-G model that best fits the given series, using ordinary least squares. To test for M-G causality *from Y to X*, a second M-G model is estimated, under the constraint $\alpha_{12}=0$. The latter equation is our *null hypothesis*. Let $\hat{\varepsilon}_t, \hat{\theta}_t$ be the residuals produced by the unconstrained and constrained best-fit M-G models, respectively. We compute and compare the sums of squared residuals $S_c = \sum_{t=1}^{N} \hat{\theta}_t$ and $S_u = \sum_{t=1}^{N} \hat{\varepsilon}_t$. Let $n_{free}=4$ be the number of free parameters in our M-G model and $n_{restr}=1$ be the number of parameters set to zero when estimating the constrained model. If the test statistic $S_F = \frac{(S_c - S_u)/n_{restr}}{S_u/(N-n_{free}-1)} \sim F_{n_{restr},N-n_{free}-1}$ is greater than a specified value, then we reject the null hypothesis that *Y* causes *X*. The p-value for the test is computed from $p = 1 - F^{cdf}_{n_{restr},N-n_{free}-1}(S_F, n_{restr}, N-n_{free}-1)$, where $F^{cdf}_{a,b}$ is the cumulative distribution function for the $F_{a,b}$ distribution.

The test requires prior selection of the parameters of the M-G process, namely τ_1, τ_2, c_1, c_2. The best delays τ_1, τ_2 are chosen on the basis of likelihood ratio tests and the Schwarz criterion. We note that our test has been constructed to filter specific non-linear structures, namely M-G. Our focus on M-G is due to the flexibility of the model and its ability to mimic properties of real series. Some examples can be found in Kyrtsou and Terraza (2003), Kyrtsou and Serletis (2006), where a noisy variant of the M-G model can filter not only its own structures but also various other non-linear effects that generate or reinforce anomalies in markets.

2. Results

To illustrate the performance of our test when it comes to capturing forms of causality that are ignored by the linear Granger test, we apply it to two simulated bivariate processes. The first process contains dynamics produced by different M-G models. The second process is chaotic variant whose properties are different from those of the M-G chaotic model. The challenge here is to see if complex interdependences can be recognised by our test no mater what the model used for creating these structures. The non-linear causality test was implemented in MATLAB[1].

[1] The code is available from the authors upon request.

The first process, originally presented in Kyrtsou (2005), was:

$$X_t=Z_t+R_t+e_t \quad e_t\sim N(0,1) \qquad (2)$$
$$Y_t=W_t+R_t+s_t \quad u_t\sim N(0,1),$$

where $Z_t = \alpha \frac{Z_{t-1}}{1+Z_{t-1}^{10}} - \delta Z_{t-1}$, $W_t= \alpha \frac{W_{t-1}}{1+W_{t-1}^{30}} - \delta W_{t-1}$, and $R_t = \alpha \frac{R_{t-1}}{1+R_{t-1}^{2}} - \delta R_{t-1}$ are three discrete-time M-G processes (Kyrtsou and Terraza (2003)). We set $Z_0=W_0=R_0=1.2$. The parameters α and δ were equal to 2.1 and 0.05 respectively. After a transient period of 1000 time steps, a pair of time series with 4096 observations was obtained. In this system, the series have a common non-linear factor, so we should expect to identify non-linear bi-directional causality.
The second process includes a mixture of stochastic and chaotic terms:

$$M_t=P_t+G_t+\varepsilon_t \quad \varepsilon_t\sim N(0,1) \qquad (3)$$
$$P_t= \varphi_1 P_{t-1}+ u_t \quad u_t\sim N(0,1).$$

Here, P_t is an AR(1) process, $G_t=4G_{t-1}(1- G_{t-1})$ is the logistic equation, and $P_o=1.2$, $G_o=0.2$. By construction, the only (linear) causal relationship to be detected should be from P to M.

TABLE 1. TEST FOR NON-LINEAR CAUSALITY

	Relation	F statistic	Probability*	Lags for M-G
X,Y	X→Y	62.4593	3.4e-015	$\tau_1=\tau_2=1$, $c_1=10$, $c_2=2$
	Y→X	140.7679	0.000	
M,P	M→P	0.5824	0.4454	$\tau_1=\tau_2=1$, $c_1=c_2=2$
	P→M	0.5598	0.4544	

* If prob>0.05, then at 5%we accept the H_0 that A does not cause B.

As Table 1 shows, in the case of series X and Y our test correctly suggests bi-directional non-linear causality (prob.<0.05). This finding is in line with the theoretical assumptions driving the simulated series X and Y. We chose $\tau_1=\tau_2=1$ to correspond with the delays present in the models (2) and (3). Similarly, the absence of non-liner causality between M and P is clearly detected using our non-linear modification of the Granger test. The probabilities are widely superior to 0.05.

CONCLUSIONS

This paper proposed a test procedure that attempts to elucidate the nature of the causality that might be obtained between time series with chaotic components. The test's main advantage is that it can be effective even in the presence of dynamics that are not generated by the M-G model but by a combination of linear stochastic and chaotic series. Of course, before applying the test it is necessary to undertake some preliminary study of the dynamical properties of the series to be tested. Further work is needed to describe the class of dynamics that our method is able to detect. Nevertheless, prior detection of the proposed theoretical structures (2) and (3) in real series suggests consistency of the resulting modified F statistics.

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